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BBA | Semester 6 | UK6DSCMGT303

Statistics for Business Decisions

Course Code: UK6DSCMGT303 • Module 5 Notes

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Module 5: Time Series Analysis

Components, Decomposition Models, Moving Averages, Least Squares Fitting, and Sales Forecasting

Chapter 1: The Epistemological Nature and Purpose of Time Series Analysis

1.1 Definitional Framework and Chronological Sequences

In contemporary enterprise administration, business conditions are never static; economic variables evolve dynamically across chronological time. Corporate sales fluctuate from month to month; commodity raw material prices drift over quarters; national gross domestic product expands over decades; and stock market equity indices oscillate daily. A statistical dataset compiled by recording quantitative observations of a specific commercial variable at successive, chronological, equal intervals of time is termed a **Time Series**.

According to Morris Hamburg, "A time series is a set of statistical observations arranged in chronological order." Croxton and Cowden formulate: "A time series consists of data which are chronologically arranged." Ya-lun Chou defines time series as "a sequence of values of some variable at different successive periods of time."

Mathematically, a time series is represented as a functional pairing between time (t) and the observed numerical magnitude of the variable (Y_t):

> $$Y = f(t) \quad \text{where } t \text{ represents time (days, weeks, months, quarters, or years)}$$
>
> **Chronological Distinctions:** Unlike cross-sectional data—which captures a single snapshot of diverse economic units at one fixed point in time (e.g., examining sales across 50 branches in December 2024)—time series data tracks the longitudinal trajectory of a single economic unit across chronological history.

1.2 Managerial Significance of Time Series Analysis

The systematic decomposition and modeling of time series data fulfills four critical executive mandates:

1. **Understanding Historical Trajectory and Dynamics:** By dissecting past performance records, management evaluates the fundamental direction of enterprise growth, identifying past operational successes, cyclical vulnerabilities, and seasonal bottlenecks.
2. **Formulating Scientific Forecasts for Future Planning:** Modern supply chains, production facilities, cash flow schedules, and workforce hiring plans depend upon reliable forecasts of future consumer demand. Time series analysis isolates underlying trends to project operational requirements over upcoming fiscal quarters.
3. **Facilitating Comparative Performance Benchmarking:** By filtering out seasonal spikes and cyclical swings, time series decomposition reveals the true underlying secular growth rate,

enabling fair performance comparisons between disparate operating divisions.

4. **Evaluating the Efficacy of Past Strategic Interventions:** When an enterprise launches a multi-year digital transformation initiative or an aggressive marketing campaign, time series analysis isolates whether subsequent revenue expansions reflect the strategy or are merely routine seasonal surges.

Chapter 2: The Four Structural Components of a Time Series

Classical statistical theory posits that the movements observed in any commercial time series are the combined, aggregate result of four distinct underlying operational forces, conventionally termed the **Components of a Time Series**:

2.1 Secular Trend (T)

The **Secular Trend** (or Long-Term Trend, T) represents the smooth, continuous, underlying directional movement of a time series over an extended chronological time horizon (typically spanning several decades). It reflects long-term structural forces that operate steadily in one direction—either a persistent upward growth or a steady downward contraction.

Fundamental drivers of secular trend include: (a) demographic population expansion and shifting age demographics; (b) long-term technological innovations (e.g., the historical expansion of semiconductor production and the corresponding decline of typewriter manufacturing); (c) capital accumulation and macroeconomic industrialization; and (d) evolving consumer tastes and lifestyle habits. Trend reflects long-term equilibrium, ignoring transient short-term oscillations.

2.2 Seasonal Variations (S)

Seasonal Variations (S) are rhythmic, periodic, wave-like fluctuations that repeat themselves with remarkable regularity within a single chronological year (or shorter operational cycle). The primary defining hallmark of seasonal variation is its **Known, Predictable Periodicity** (repeating every 12 months, 4 quarters, or 7 days of the week).

Seasonal variations originate from two foundational causes:

- **Natural and Climatic Forces:** Periodic weather changes directly govern human consumption and production. Air-conditioner sales and ice cream consumption surge in summer months; woolen clothing and room heater sales peak in winter; agricultural harvest yields surge following monsoon seasons.
- **Social, Cultural, and Institutional Conventions:** Cultural festivals and legal conventions trigger intense annual shopping peaks. In India, retail gold, garment, and consumer electronics sales spike during Diwali, Dussehra, and the wedding season; tax accounting and legal audit services peak in March; commercial bank transactions surge during the first week of every month.

2.3 Cyclical Fluctuations (C)

Cyclical Fluctuations (C) are oscillatory, wave-like movements that recur across extended time horizons exceeding one year (typically spanning 3 to 10 years per complete cycle). Unlike seasonal

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patterns, cyclical fluctuations do not possess a fixed, predictable mathematical duration; they reflect the recurrent macroeconomic **Business Cycle** (Trade Cycle).

A complete commercial business cycle comprises four sequential operational phases:

1. **Prosperity (Boom / Peak):** High industrial production, full employment, expanding corporate earnings, aggressive capital investments, and inflationary price pressures.
2. **Recession:** Economic activity slows; consumer demand contracts; corporate inventories accumulate; and business optimism deteriorates.
3. **Depression (Trough / Slump):** Widespread factory capacity underutilization, elevated unemployment, falling interest rates, declining corporate profits, and liquidity distress.
4. **Recovery (Expansion):** Replacement capital investments resume; consumer demand revives; employment expands; and business confidence is restored.

2.4 Irregular or Random Movements (I)

Irregular Variations (or Residual / Random Movements, I) encompass all erratic, chaotic, and completely unpredictable fluctuations that cannot be attributed to trend, season, or cycle. They originate from sudden, exogenous catastrophic shocks and unexpected operational disruptions.

Typical drivers include: (a) natural disasters (earthquakes, floods, tsunamis); (b) geopolitical conflicts and wars; (c) industrial labor strikes and unexpected factory fires; (d) sudden regulatory bans; and (e) global macroeconomic and biological pandemics (such as the COVID-19 pandemic disruptions of 2020). By definition, irregular shocks cannot be forecasted mathematically.

Chapter 3: Mathematical Models of Time Series Decomposition

To analyze time series data, statisticians mathematically formulate how the four components combine to produce the observed raw data value (Y):

3.1 The Additive Model

The **Additive Model** assumes that the observed time series value is the direct arithmetic sum of the four separate, independent components:

> $$Y_t = T_t + S_t + C_t + I_t$$
>
> **Theoretical Assumption:** The components are assumed to be mutually independent and expressed in the **identical absolute physical units of measurement** as the original data. Seasonal, cyclical, and irregular swings are treated as absolute rupee or unit deviations above or below the secular trend.

3.2 The Multiplicative Model (Standard Commercial Practice)

The **Multiplicative Model** assumes that the observed time series value is the product of the four interdependent components:

> $$Y_t = T_t \times S_t \times C_t \times I_t$$
>
> **Theoretical Assumption:** The secular trend (T_t) is expressed in absolute physical units of measurement (e.g., rupees or tonnes), while seasonal (S_t), cyclical (C_t), and irregular (I_t) components are expressed as **dimensionless relative ratios, percentages, or index multipliers** centered around 1.0 (or 100%).

The multiplicative model represents the standard choice in economics because in real-world commerce, seasonal fluctuations do not remain constant in absolute terms; as enterprise business scale expands over time, seasonal peaks expand proportionally in absolute rupee value.

Chapter 4: Methods of Measuring Secular Trend

Statisticians deploy four distinct methodologies to isolate and measure the secular trend component:

4.1 The Freehand Graphic Method

Raw time series data is plotted on a Cartesian coordinate grid with time on the X-axis and variable values on the Y-axis. The analyst visually sketches a smooth, continuous line or curve passing through the center of the plotted points, attempting to balance points above and below the line. While virtually instantaneous and requiring zero mathematics, the method is completely subjective; different analysts produce divergent trend lines for identical data.

4.2 The Semi-Averages Method

The chronological dataset is partitioned into two equal halves. The arithmetic mean of the first half is computed and plotted at the midpoint of that time segment; the arithmetic mean of the second half is computed and plotted at the midpoint of the second segment. A straight line is drawn connecting the two plotted points and extended outward to establish the trend line.

> **Odd Period Rule:** If the series comprises an odd number of periods (e.g., 7 or 9 years), the central middle year is completely omitted from calculations, and the remaining equal halves are averaged.

4.3 The Moving Averages Method

The **Moving Averages Method** is a non-parametric mathematical smoothing technique that calculates successive overlapping arithmetic means over a specified moving span or window (e.g., 3-year, 5-year, or 12-month moving averages).

1. Periodicity Selection Principle

To eliminate cyclical fluctuations completely, the span of the moving average must precisely equal the **known average duration of the cycle**. A 5-year moving average completely flattens and neutralizes a regular 5-year business cycle, isolating the pure secular trend.

2. Centering for Even-Period Moving Averages

When computing moving averages over an odd span (e.g., 3 or 5 years), the calculated average naturally centers at the exact middle year (e.g., the average of years 1, 2, and 3 is placed against year 2). However, when the moving span is an even number (e.g., 4 quarters or 12 months), the computed average falls between two time periods. To align averages with specific chronological periods, analysts compute a secondary **Centered Moving Average** (averaging two adjacent moving averages).

Merits and Demerits: Moving averages are objective and flexible, adapting to non-linear trends. However, their primary demerit is the **Loss of Boundary Data**: a 5-year moving average loses the first two and final two years of data, rendering it completely incapable of generating future forecasts.

Chapter 5: The Mathematical Method of Least Squares for Linear Trend Fitting

5.1 Definitional Architecture and the Linear Model

The **Method of Least Squares** is universally recognized as the most scientifically rigorous, objective, and mathematically robust technique for determining secular trend. It fits a continuous mathematical function to the data satisfying the least-squares error minimization criteria.

The standard linear secular trend line is formulated as:

> $$Y_t = a + bX$$
>
> **Where:** Y_t represents the estimated trend value at time period t;
> X represents the chronological time variable (measured as deviations from a chosen origin);
> a represents the trend value at the origin ($X = 0$);
> b represents the constant annual rate of trend growth (slope).

5.2 The Normal Equations and Simplified Computational Short-Cuts

The parameters a and b are determined through the simultaneous solution of two standard normal equations:

> Normal Equation 1: $\sum Y = Na + b\sum X$
>
> Normal Equation 2: $\sum XY = a\sum X + b\sum X^2$

In practice, management accountants employ an elegant computational short-cut by setting the **Origin at the Exact Middle Point of Time**, which forces the sum of the time deviations to equal zero ($\sum X = 0$\$). Under this condition, the normal equations simplify into two independent, instantaneous formulas:

> When $\sum X = 0$: $\quad a = \frac{\sum Y}{N} \quad$ and $\quad b = \frac{\sum XY}{\sum X^2}$

> **Algebraic Beauty:** Parameter a is simply the arithmetic mean of all historical observations ($\bar{Y}$), while parameter b directly represents the annual slope increment.

5.3 Setting the Origin for Odd versus Even Number of Periods

Case 1: Odd Number of Years (e.g., N = 5 or 7 years)

The origin ($X = 0$\$) is established at the exact chronological middle year. Time deviations (X\$) are assigned in natural integer steps: $-3, -2, -1, 0, +1, +2, +3$. Here, one unit of X equals exactly one calendar year, and $\sum X = 0$.

Case 2: Even Number of Years (e.g., N = 6 or 8 years)

The origin is established midway between the two central chronological years. To eliminate fractions, deviations are measured in **half-year units** (half-year $= 0.5$ year\$), assigning odd integer values: $-5, -3, -1, +1, +3, +5$. Under this convention, one unit of X represents half a year, and $\sum X = 0$.

> **Critical Forecasting Note for Even Years:** Because one unit of X represents 0.5 year, the calculated slope parameter b represents the *semi-annual rate of growth*. The annual rate of growth is equal to $2b$.

Chapter 6: Measurement of Seasonal Variations and Forecasting Applications

6.1 The Ratio-to-Moving-Average Method

The **Ratio-to-Moving-Average Method** is the most widely deployed commercial technique for isolating seasonal variations. It operates across four systematic stages under the multiplicative model ($Y = T \times S \times C \times I$):

1. **Compute Centered 12-Month (or 4-Quarter) Moving Averages:** This smooths out seasonal variations and irregular noise, isolating the Trend-Cycle component (TC\$).
2. **Calculate Percentage Ratios of Raw Data to Moving Averages:** For each period, the original raw data (Y\$) is divided by the moving average (TC\$):

   $$\frac{Y}{TC} \times 100 = \frac{T \times S \times C \times I}{TC} \times 100 = S \times I$$

   This isolates the combined Seasonal and Irregular components (SI\$).
3. **Eliminate Irregular Shocks via Monthly / Quarterly Averaging:** The SI ratios for identical calendar months (e.g., all Januaries across multiple years) are averaged using the median or a modified mean, eliminating random irregular noise and isolating pure **Unadjusted Seasonal Indices** (S\$).

4. **Normalize to a 100% Grand Average:** The unadjusted indices are scaled proportionally so that the sum of the four quarterly indices equals 400 (or the sum of twelve monthly indices equals 1,200), establishing the final **Seasonal Index**.

6.2 Deseasonalization of Data

To evaluate authentic underlying business performance, raw data is purged of seasonal distortion through **Deseasonalization**:

$$\text{Deseasonalized Value} = \frac{\text{Actual Raw Value}}{\text{Seasonal Index}} \times 100$$

Managerial Utility: If a retail brand's sales expand by 15% in Diwali month, but the seasonal index indicates a normal 25% seasonal surge, the deseasonalized figure reveals that underlying demand actually contracted relative to seasonal norms.

Chapter 7: Comprehensive Worked Practical Problems in Time Series Analysis

7.1 Worked Problem 1: Method of Least Squares Linear Trend (Odd Number of Years)

The annual sales revenue of Apex Pharmaceuticals Ltd. (in crores of rupees) over a seven-year operating horizon is presented below:

Year	Sales Y (Rs. Cr)	X = Year - 2020	X^2	X · Y	Trend Value Y_t = 30 + 3X
2017	20	-3	9	-60	30 + 3(-3) = 21
2018	23	-2	4	-46	30 + 3(-2) = 24
2019	28	-1	1	-28	30 + 3(-1) = 27
2020	31	0 (Origin)	0	0	30 + 3(0) = 30
2021	32	+1	1	+32	30 + 3(+1) = 33
2022	36	+2	4	+72	30 + 3(+2) = 36
2023	40	+3	9	+120	30 + 3(+3) = 39
Total	**Sum Y = 210**	**Sum X = 0**	**Sum X^2 = 28**	**Sum XY = +90**	**Sum Y_t = 210**

Required:

1. Fit a linear trend line by the Method of Least Squares.
2. Calculate individual annual trend values for each year from 2017 to 2023.

3. Estimate forecasted sales revenue for the years 2025 and 2026.
4. Calculate the annual rate of trend growth.

Step-by-Step Mathematical Resolution:

1. Fit the Linear Trend Line:

> Total observations $N = 7$ (Odd number of years). We set the Origin at the middle year **2020**. Therefore, $\sum X = 0$.
>
> $a = \frac{\sum Y}{N} = \frac{210}{7} = 30.00$ crores
>
> $b = \frac{\sum XY}{\sum X^2} = \frac{90}{28} = 3.214 \approx 3.21$ crores per annum
>
> Fitted Linear Trend Equation: $Y_t = 30.00 + 3.214X$ (Origin: 2020, X unit = 1 year)

2. Compute Trend Values for Historical Years:

- 2017 ($X = -3$): $Y_t = 30 + 3.214(-3) = 30 - 9.642 = 20.36$ crores
- 2018 ($X = -2$): $Y_t = 30 + 3.214(-2) = 30 - 6.428 = 23.57$ crores
- 2019 ($X = -1$): $Y_t = 30 + 3.214(-1) = 30 - 3.214 = 26.79$ crores
- 2020 ($X = 0$): $Y_t = 30 + 3.214(0) = 30.00$ crores
- 2021 ($X = +1$): $Y_t = 30 + 3.214(+1) = 33.21$ crores
- 2022 ($X = +2$): $Y_t = 30 + 3.214(+2) = 36.43$ crores
- 2023 ($X = +3$): $Y_t = 30 + 3.214(+3) = 39.64$ crores

> **Verification:** $\sum Y_t = 20.36 + 23.57 + 26.79 + 30.00 + 33.21 + 36.43 + 39.64 = 210.00 = \sum Y$. Perfect mathematical harmony.

3. Forecast Future Sales for 2025 and 2026:

> For 2025: $X = 2025 - 2020 = +5 \implies Y_{2025} = 30.00 + 3.214(5) = 30.00 + 16.07 = 46.07$ crores
>
> For 2026: $X = 2026 - 2020 = +6 \implies Y_{2026} = 30.00 + 3.214(6) = 30.00 + 19.28 = 49.28$ crores

7.2 Worked Problem 2: Method of Least Squares Linear Trend (Even Number of Years)

The annual export volume of a marine logistics firm (in thousands of metric tonnes) across six consecutive operating years is recorded as follows:

Year	Export Y (000 tonnes)	Time Deviation X (Half-Years from mid-2020.5)	X^2	X · Y
2018	50	-5	25	-250
2019	54	-3	9	-162
2020	58	-1	1	-58
2021	64	+1	1	+64
2022	68	+3	9	+204
2023	74	+5	25	+370
Total	**Sum Y = 368**	**Sum X = 0**	**Sum X^2 = 70**	**Sum XY = +168**

Step-by-Step Mathematical Resolution:

1. Parameter Determination:

Total observations $N = 6$ (Even number of years). We place the origin at the midpoint between 2020 and 2021 (mid-2020.5). Units of X are **half-years**. $\sum X = 0$.

$$a = \frac{\sum Y}{N} = \frac{368}{6} = 61.333 \text{ thousand tonnes}$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{168}{70} = 2.40 \text{ thousand tonnes per half-year}$$

$$\text{Annual Rate of Growth} = 2 \times b = 2 \times 2.40 = 4.80 \text{ thousand tonnes per annum}$$

$$\text{Trend Equation: } Y_t = 61.333 + 2.40X \quad (\text{Origin: mid-2020.5, X unit} = 0.5 \text{ year})$$

2. Forecast Future Export Volume for 2025:

$$\text{Years from mid-2020.5 to mid-2025} = 2025.5 - 2020.5 = 4.5 \text{ years}$$

$$X \text{ units (in half-years)} = 4.5 \times 2 = +9$$

$$Y_{2025} = 61.333 + 2.40(9) = 61.333 + 21.60 = 82.933 \approx 82.93 \text{ thousand tonnes}$$

Executive Strategic Plan: Port warehousing capacity must expand to accommodate approximately **82,933 metric tonnes** by 2025 based on secular least-squares trend modeling.

7.3 Worked Problem 3: 3-Yearly and 5-Yearly Moving Average Computation

The annual sales of an industrial valve supplier over nine consecutive operating years are presented below:

Year	Sales (Rs. Lakhs)	3-Year Moving Total	3-Year Moving Average (Trend)	5-Year Moving Total	5-Year Moving Average (Trend)
2015	12	-	-	-	-
2016	15	45 (12+15+18)	15.00	-	-
2017	18	51 (15+18+18)	17.00	87	17.40
2018	18	60 (18+18+24)	20.00	99	19.80
2019	24	66 (18+24+24)	22.00	114	22.80
2020	24	78 (24+24+30)	26.00	132	26.40
2021	30	87 (24+30+33)	29.00	153	30.60
2022	33	105 (30+33+42)	35.00	-	-
2023	42	-	-	-	-

Managerial Insights from Moving Average Curves:

1. **Smoothing Efficacy:** Both the 3-year and 5-year moving averages successfully filter out minor irregular fluctuations, revealing an uninterrupted, strong upward secular growth trend from Rs. 15.00 lakhs to Rs. 35.00 lakhs.
2. **Span Comparison:** The 5-year moving average creates a noticeably smoother trajectory than the 3-year curve, but it suffers from greater boundary data loss (losing 4 total years versus only 2 years lost in the 3-year moving average).

7.4 Worked Problem 4: 4-Quarterly Centered Moving Averages and Seasonal Indices

A regional consumer apparel brand records quarterly revenue (in lakhs of rupees) across two consecutive operating years:

Year	Quarter	Revenue Y	4-Quarter Moving Total	4-Quarter Centered Total	Centered Moving Average (Trend TC)	Ratio to Trend (Y / TC) · 100
Year 1	Q1	60	-	-	-	-
Year 1	Q2	80	-	-	-	-

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Year	Quarter	Revenue Y	4-Quarter Moving Total	4-Quarter Centered Total	Centered Moving Average (Trend TC)	Ratio to Trend (Y / TC) · 100
Year 1	Q3	70	310 (60+80+70+100)	630 (310+320)	78.75	88.89%
Year 1	Q4	100	320 (80+70+100+70)	650 (320+330)	81.25	123.08%
Year 2	Q1	70	330 (70+100+70+90)	670 (330+340)	83.75	83.58%
Year 2	Q2	90	340 (100+70+90+80)	690 (340+350)	86.25	104.35%
Year 2	Q3	80	350 (70+90+80+110)	-	-	-
Year 2	Q4	110	-	-	-	-

Step-by-Step Computational Protocol:

> Centered Moving Average for Year 1 Q3 $= \frac{310+320}{8} = \frac{630}{8} = 78.75$
>
> Seasonal Ratio for Year 1 Q3 $= \frac{70}{78.75} \times 100 = 88.89\%$
>
> **Interpretation:** Quarter 4 exhibits an intense positive seasonal index of **123.08%** (reflecting year-end holiday and festival shopping surges), whereas Quarter 1 drops to **83.58%** during post-holiday retail lulls.

Chapter 8: Managerial Decision Framework for Demand and Inventory Forecasting

8.1 Measuring Forecast Accuracy and Error Metrics

Modern supply chain managers evaluate statistical forecasting models using three standard quantitative error metrics:

1. **Mean Absolute Deviation (MAD):** Measures average forecasting error magnitude without regard to sign:

> $$MAD = \frac{\sum |Y - Y|}{n}$$

> Directly guides safety stock sizing: $\text{Safety Stock} = z \times \sqrt{\frac{\pi}{2}} \times MAD \approx 1.25 \cdot z \cdot MAD$.

2. **Mean Squared Error (MSE):** Heavily penalizes large forecasting mistakes by squaring individual errors:

> $$MSE = \frac{\sum (Y - Y)^2}{n}$$

3. **Mean Absolute Percentage Error (MAPE):** Expresses forecast error as a percentage of actual sales, enabling performance comparisons across disparate product categories:

> $$MAPE = \frac{1}{n} \sum \left| \frac{Y - Y}{Y} \right| \times 100$$
>
> A MAPE below 10% is rated as highly accurate, while 10%–20% is acceptable.

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