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Bachelor of Business Administration (BBA) | Fifth Semester | UK5DSCMGT303

Management Science

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Module 4: Transportation Problems

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Chapter 1: Theoretical Framework and Mathematical Structure of Transportation Problems

1.1 Physical Distribution Logistics and the Transportation Paradigm

The Transportation Problem is a specialized class of linear programming models designed to determine the most cost-effective physical distribution schedule for transporting a single homogeneous commodity from a set of m supply origins (such as manufacturing factories, processing mills, or central distribution warehouses) to a set of n consuming destinations (such as regional sales depots, retail outlets, or construction sites). The overriding managerial objective is to satisfy all destination demands without exceeding origin supply capacities while minimizing the aggregate physical freight and logistics expenditure.

First formulated mathematically by Frank L. Hitchcock in 1941 and independently extended by Tjalling C. Koopmans in 1947, the transportation model represents the analytical backbone of global supply chain management, intermodal freight logistics, maritime container routing, and petroleum pipeline distribution.

1.2 Mathematical Formulation of the Transportation Model

Let an enterprise possess m supply origins, with origin i having an available capacity of ai units, and n demand destinations, with destination j requiring bj units. Let cij represent the unit freight cost of shipping one unit of commodity from origin i to destination j$, and let xij denote the continuous decision variable representing the physical quantity shipped along that specific route. The general linear programming formulation is:

> Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$
>
> Subject to structural constraints:
> 1. Origin Supply Constraints: $\sum_{j=1}^{n} x_{ij} \le a_i$ (for each origin $i = 1, 2, \dots, m$)
> 2. Destination Demand Constraints: $\sum_{i=1}^{m} x_{ij} \ge b_j$ (for each destination $j = 1, 2, \dots, n$)
> 3. Non-negativity Restrictions: $x_{ij} \ge 0$ (for all i, j)

Legend: Z = Aggregate transportation expenditure; c_{ij} = Unit freight cost from origin i to destination j; x_{ij} = Quantity shipped from origin i to destination j; a_i = Total supply available at origin i; b_j = Total demand required at destination j.

1.3 Balanced versus Unbalanced Transportation Models

A transportation problem is classified as **Balanced (Rim-Balanced)** if total available supply across all origins exactly equals total required demand across all destinations:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Legend: Rim requirement balance condition ensuring all supply constraints and demand constraints hold as strict equalities.

When aggregate supply and aggregate demand are not equal ($\sum a_i \neq \sum b_j$), the problem is **Unbalanced**. Standard transportation algorithms require rim parity before execution. Balancing is achieved by introducing fictitious dummy facilities:

- **Excess Supply Condition ($\sum a_i > \sum b_j$):** Total production exceeds customer demand. Add a **Dummy Destination (D_{n+1})** with requirement $b_{n+1} = \sum a_i - \sum b_j$ and unit shipping costs of **Rs 0** ($c_{i,n+1} = 0$). Allocations made to this dummy destination represent inventory that remains unsold in factory warehouses.
- **Excess Demand Condition ($\sum a_i < \sum b_j$):** Market demand exceeds total factory output. Add a **Dummy Origin (O_{m+1})** with capacity $a_{m+1} = \sum b_j - \sum a_i$ and unit shipping costs of **Rs 0** ($c_{m+1,j} = 0$). Allocations made from this dummy origin represent unmet customer shortages or backorders.

1.4 Fundamental Properties of Basic Feasible Solutions

A basic feasible solution (BFS) to an $m \times n$ balanced transportation problem exhibits two fundamental mathematical properties:

1. **Exact Number of Basic Allocations ($m + n - 1$):** Because the rim equality condition ($\sum a_i = \sum b_j$) introduces one redundant constraint into the system of $m + n$ equations, exactly $m + n - 1$ decision variables must be basic (positive allocations $x_{ij} > 0$), while the remaining $m \cdot n - (m + n - 1$ non-basic variables are zero.
2. **Independence of Allocations (Loopless Condition):** The allocated basic cells must occupy independent positions in the transportation tableau such that it is mathematically impossible to form a closed continuous loop consisting of alternating horizontal and vertical line segments turning only at allocated cells.

Chapter 2: Methods for Finding an Initial Basic Feasible Solution (IBFS)

2.1 Comparison of the Three Classical IBFS Heuristics

To initiate optimization, corporate planners must generate a starting Initial Basic Feasible Solution (IBFS). Three standardized heuristics are deployed in practice:

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Heuristic Method	Core Algorithmic Logic	Managerial Trade-offs & Performance
1. North West Corner Rule (NWCR)	Purely positional / geometric heuristic. Begins at top-left cell (1, 1), allocates maximum feasible quantity, exhausts supply or demand, steps right or down.	Completely ignores shipping cost coefficients (c_{ij}). Computationally trivial to execute manually, but yields an initial solution far removed from global optimality with high freight costs.
2. Lowest Cost Method (LCM / Matrix Minimum)	Greedy economic heuristic. Searches the entire cost matrix for the cell with the absolute lowest unit cost ($\min c_{ij}$), allocates maximum feasible quantity, crosses out satisfied row/col, and repeats.	Directly exploits cost differentials. Produces significantly lower starting logistics costs than NWCR, but can inadvertently force massive allocations into high-cost cells in late iterations.
3. Vogel's Approximation Method (VAM)	Unit penalty / opportunity regret heuristic. Computes the penalty difference between the two lowest costs in each row and column, selects the vector with largest penalty, and allocates to its minimum-cost cell.	Best-in-class heuristic. Captures the opportunity cost of failing to allocate to the best route. In over 80% of industrial cases, VAM yields an initial solution that is already global optimal or within 2% of optimal!

2.2 The Detailed Vogel's Approximation Method (VAM) Algorithm

Because VAM is the industry standard for professional operations research, its algorithmic execution is formalized in four iterative steps:

- **Step 1: Compute Row and Column Penalties:** For every active row, find the smallest and second smallest unit shipping costs; compute their positive difference ($P_i = c_{i,second} - c_{i,first}$). Repeat the exact same computation for every active column (P_j).
- **Step 2: Identify Maximum Penalty:** Scan all computed row and column penalties. Identify the row or column possessing the largest penalty. If a tie occurs, break it by selecting the vector containing the absolute lowest unit cost.
- **Step 3: Allocate Maximum Feasible Quantity:** In the selected vector, locate the cell with the lowest unit cost (c_{ij}). Allocate the maximum permissible quantity:

```
xij = min(ai, bj)
```

Legend: x_{ij} = Quantity allocated; a_i = Available supply at origin i; b_j = Unsatisfied demand at destination j.

- **Step 4: Update Rim Requirements and Eliminate Satisfied Vectors:** Deduct xij from both origin capacity and destination requirement. Cross out the row if supply is exhausted, or cross out the column if demand is satisfied. If both supply and demand are satisfied simultaneously, cross out only one vector and assign a basic zero (ϵ) to an uncrossed cell to prevent degeneracy. Recompute penalties for remaining active cells until all demands are met.

Chapter 3: Optimality Testing and the Modified Distribution (MODI) Method

3.1 Mathematical Theory of Optimality (The u-v Method)

Once an IBFS containing exactly m + n – 1 independent allocations is established, management must verify whether that solution is globally optimal or whether freight costs can be further reduced. The standard diagnostic engine is the **Modified Distribution Method (MODI)**, which exploits linear programming duality:

To each supply origin i$, associate a dual variable ui$; to each demand destination j$, associate a dual variable vj$. These dual variables are determined by enforcing the condition that for all **Allocated (Basic) Cells**:

```
ui + vj = cij  (for all allocated cells with xij > 0)
```

Legend: u_i = Dual potential of origin i; v_j = Dual potential of destination j; c_{ij} = Actual unit cost of allocated route.

To solve the system of m + n – 1 equations containing m + n variables, arbitrarily set one dual variable equal to zero (conventionally, let u1 = 0) and solve sequentially for all remaining ui and vj potentials.

3.2 Net Evaluation Index (Opportunity Cost) for Empty Cells

Next, for every **Unallocated (Empty) Cell**, calculate the Opportunity Cost (Net Evaluation Index Δij):

```
Δij = cij – (ui + vj)
```

Legend: \Delta_{ij} = Net evaluation index / opportunity cost; c_{ij} = Actual direct unit cost of shipping along empty route; (u_i + v_j) = Imputed theoretical dual cost.

The Global Optimality Criterion:

- If **all Δij ≥ 0** for all unallocated cells, the current transportation schedule is **GLOBALLY OPTIMAL**! No cheaper routing exists, and total freight cost is minimized.
- If any **Δij = 0**, alternative equally optimal solutions exist, giving management routing flexibility.
- If **one or more Δij < 0**, the current solution is suboptimal. Allocating freight into a cell with negative Δij will reduce total distribution costs by |Δij| for every unit shipped!

3.3 Solution Improvement via Closed Stepping-Stone Loops

If the optimality test fails, improve the solution via the Stepping Stone loop procedure:

1. **Identify Entering Cell:** Select the empty cell possessing the most negative evaluation index ($\min \Delta ij < 0$).
2. **Construct Closed Evaluation Loop:** Trace a closed geometric loop starting and ending at the entering cell. The loop must consist exclusively of alternating horizontal and vertical line segments, and all turning corners must land strictly on currently allocated basic cells.
3. **Assign Alternating Signs (+, −):** Assign a positive sign (+) to the entering cell, and alternating negative (−) and positive (+) signs to the successive corners of the loop.
4. **Determine Reallocation Quantity (θ):** Locate all loop corners marked with a negative sign (−). Find the minimum allocation among these donor cells:

   ```
   θ = min{xij | corner (i, j) has a minus sign}
   ```

   Legend: \theta = Maximum permissible quantity of freight that can be shifted around the closed loop without violating non-negativity.
5. **Shift Allocations:** Add θ to all positive corners, and subtract θ from all negative corners. The donor cell with allocation reduced to zero leaves the basic set, while the entering cell joins the basic set. Re-apply MODI optimality testing to the revised schedule.

Chapter 4: Degeneracy and Special Complications in Transportation

4.1 Concept and Resolution of Degeneracy

Degeneracy occurs in a transportation problem whenever the number of basic allocations is strictly less than $m + n - 1$. In a degenerate tableau, it is mathematically impossible to compute all dual variables (u_i, v_j) because the system of equations becomes undetermined.

Degeneracy arises in two operational contexts:

- **Initial Degeneracy:** Occurs during IBFS generation when a single allocation simultaneously satisfies both an origin supply and a destination demand, eliminating both a row and a column in a single step.
- **Intermediate Degeneracy:** Occurs during loop reallocation when two or more negative corners tie for the minimum allocation θ, causing multiple cells to become zero simultaneously.

The Epsilon (ϵ) Resolution Protocol: To resolve degeneracy, allocate an infinitesimal, negligibly small positive quantity ($\epsilon > 0$) to an unallocated cell that has a very low unit cost and does not form a closed loop with existing basic cells. The quantity ϵ is treated as a positive basic variable during dual variable calculations and loop formulations, but is set equal to zero ($\epsilon \to 0$) when computing physical freight quantities and accounting costs.

4.2 Prohibited Routes and Transshipment

When physical blockages (such as low-clearance bridges, political border embargos, or hazardous materials bans) prohibit shipping along route (i, j), assign an extraordinarily high cost penalty $M \to \infty$ to that cell, preventing any allocation.

The **Transshipment Model** extends basic transportation by recognizing that goods can move between origins, through intermediate consolidation hubs (cross-docks), and between destinations prior to final customer delivery. It is solved by adding buffer capacities equal to total system volume to all transshipment nodes.

Chapter 5: Comprehensive Worked Numerical Problem

5.1 Problem Statement: Three Factories to Four Warehouses

A consumer goods corporation produces packaged foods across three manufacturing plants—**P1, P2, P3**—and distributes goods to four regional warehouses—**W1, W2, W3, W4**. The plant monthly capacities, warehouse requirements, and per-unit freight costs (in Rupees per case) are given in the transportation matrix:

Manufacturing Plant	W1	W2	W3	W4	Plant Capacity (a_i)
Plant P1	Rs 19	Rs 30	Rs 50	Rs 10	**7 cases**
Plant P2	Rs 70	Rs 30	Rs 40	Rs 60	**9 cases**
Plant P3	Rs 40	Rs 8	Rs 70	Rs 20	**18 cases**
Warehouse Demand (b_j)	**5 cases**	**8 cases**	**7 cases**	**14 cases**	**Total = 34 cases**

Rim Parity Verification:

Total Supply = 7 + 9 + 18 = 34 cases. Total Demand = 5 + 8 + 7 + 14 = 34 cases. Total Supply = Total Demand = 34 cases. The problem is perfectly balanced! Basic allocations required = m + n − 1 = 3 + 4 − 1 = 6 basic cells.

5.2 Initial Solution Generation via Vogel's Approximation Method (VAM)

Let us compute row and column penalty differences (two smallest costs):

- **Iteration 1:**
  - Row Penalties: P1 = |19 - 10| = 9; P2 = |40 - 30| = 10; P3 = |20 - 8| = 12.
  - Column Penalties: W1 = |40 - 19| = 21; W2 = |30 - 8| = 22; W3 = |50 - 40| = 10; W4 = |20 - 10| = 10.

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- Maximum penalty is **22 in Column W2**! Smallest cost in Col W2 is Rs 8 at cell (P3, W2).
- Allocate maximum feasible: $x32 = \min(a3 = 18, b2 = 8) = 8$ cases. Warehouse W2 demand satisfied; cross out Column W2. Plant P3 remaining capacity = 18 - 8 = 10 cases.

- **Iteration 2:**
  - Row Penalties: P1 = |19 - 10| = 9; P2 = |60 - 40| = 20; P3 = |40 - 20| = 20.
  - Column Penalties: W1 = |40 - 19| = 21; W3 = |50 - 40| = 10; W4 = |20 - 10| = 10.
  - Maximum penalty is **21 in Column W1**! Smallest cost in Col W1 is Rs 19 at cell (P1, W1).
  - Allocate maximum feasible: $x11 = \min(a1 = 7, b1 = 5) = 5$ cases. Warehouse W1 satisfied; cross out Col W1. Plant P1 remaining capacity = 7 - 5 = 2 cases.
- **Iteration 3:**
  - Remaining columns: W3 (demand 7) and W4 (demand 14).
  - Row Penalties: P1 = |50 - 10| = 40; P2 = |60 - 40| = 20; P3 = |70 - 20| = 50.
  - Column Penalties: W3 = |50 - 40| = 10; W4 = |20 - 10| = 10.
  - Maximum penalty is **50 in Row P3**! Smallest cost in Row P3 is Rs 20 at cell (P3, W4).
  - Allocate: $x34 = \min(a3 = 10, b4 = 14) = 10$ cases. Plant P3 capacity exhausted; cross out Row P3. Warehouse W4 remaining demand = 14 - 10 = 4 cases.
- **Iteration 4:**
  - Remaining cells: (P1, W4) with cost 10; (P1, W3) with cost 50; (P2, W3) with cost 40; (P2, W4) with cost 60.
  - Plant P1 has 2 cases remaining; allocate to cheapest remaining cell: x14 = 2 cases to (P1, W4). P1 capacity exhausted; cross out Row P1. Remaining W4 demand = 4 - 2 = 2 cases.
  - Plant P2 has 9 cases remaining; allocate 2 cases to (P2, W4) to satisfy W4: x24 = 2.
  - Allocate remaining 7 cases of P2 to (P2, W3): x23 = 7. Warehouse W3 demand completely satisfied!

Initial VAM Allocation Schedule

Origin Plant	Destination Warehouse	Allocated Units	Unit Cost & Freight Total
Plant P1	Warehouse W1	5 cases	5 × Rs 19 = Rs 95
Plant P1	Warehouse W4	2 cases	2 × Rs 10 = Rs 20
Plant P2	Warehouse W3	7 cases	7 × Rs 40 = Rs 280
Plant P2	Warehouse W4	2 cases	2 × Rs 60 = Rs 120
Plant P3	Warehouse W2	8 cases	8 × Rs 8 = Rs 64

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Origin Plant	Destination Warehouse	Allocated Units	Unit Cost & Freight Total
Plant P3	Warehouse W4	10 cases	10 × Rs 20 = Rs 200
Total	**Initial Basic Feasible Schedule**	**34 cases**	**Rs 779 (Initial VAM Cost)**

Total allocations = 6 basic cells (m + n − 1 = 6). The solution is non-degenerate and ready for MODI testing!

5.3 MODI Optimality Testing and Final Global Optimization

Set $u1 = 0$ and solve $ui + vj = cij$ for all basic cells:

- Cell (P1, W1): $u1 + v1 = 19 \implies 0 + v1 = 19 \implies v1 = 19$
- Cell (P1, W4): $u1 + v4 = 10 \implies 0 + v4 = 10 \implies v4 = 10$
- Cell (P2, W4): $u2 + v4 = 60 \implies u2 + 10 = 60 \implies u2 = 50$
- Cell (P2, W3): $u2 + v3 = 40 \implies 50 + v3 = 40 \implies v3 = -10$
- Cell (P3, W4): $u3 + v4 = 20 \implies u3 + 10 = 20 \implies u3 = 10$
- Cell (P3, W2): $u3 + v2 = 8 \implies 10 + v2 = 8 \implies v2 = -2$

Opportunity Cost Calculation for Unallocated Cells: $\Delta ij = cij - (ui + vj)$

- Cell (P1, W2): $\Delta 12 = 30 - (0 + -2) = 30 - (-2) = +32 \geq 0$
- Cell (P1, W3): $\Delta 13 = 50 - (0 + -10) = 50 - (-10) = +60 \geq 0$
- Cell (P2, W1): $\Delta 21 = 70 - (50 + 19) = 70 - 69 = +1 \geq 0$
- Cell (P2, W2): $\Delta 22 = 30 - (50 + -2) = 30 - 48 = -18 < 0$ (Suboptimal! Negative index indicates improvement opportunity)
- Cell (P3, W1): $\Delta 31 = 40 - (10 + 19) = 40 - 29 = +11 \geq 0$
- Cell (P3, W3): $\Delta 33 = 70 - (10 + -10) = 70 - 0 = +70 \geq 0$

Stepping Stone Loop Reallocation

The entering cell is **(P2, W2)** with $\Delta 22 = -18$. We form a closed loop around allocated cells:

Loop: **(P2, W2) [+] → (P2, W4) [-] → (P3, W4) [+] → (P3, W2) [-] → back to (P2, W2)**

Negative donor corners are (P2, W4) with 2 cases, and (P3, W2) with 8 cases. Minimum allocation is $\theta = \min(2, 8) = 2$ cases!

- Cell (P2, W2): 0 + 2 = **2 cases** (Enters basis)
- Cell (P2, W4): 2 - 2 = **0 cases** (Leaves basis)

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- Cell (P3, W4): 10 + 2 = **12 cases**
- Cell (P3, W2): 8 - 2 = **6 cases**
- All other allocated cells (P1-W1 = 5, P1-W4 = 2, P2-W3 = 7) remain unchanged!

Final Global Optimal Schedule and Audited Minimum Cost

Origin Plant	Destination Warehouse	Allocated Units	Freight Expenditure
Plant P1	Warehouse W1	5 cases	5 × Rs 19 = Rs 95
Plant P1	Warehouse W4	2 cases	2 × Rs 10 = Rs 20
Plant P2	Warehouse W2	2 cases	2 × Rs 30 = Rs 60
Plant P2	Warehouse W3	7 cases	7 × Rs 40 = Rs 280
Plant P3	Warehouse W2	6 cases	6 × Rs 8 = Rs 48
Plant P3	Warehouse W4	12 cases	12 × Rs 20 = Rs 240
Total	**Global Optimal Distribution**	**34 cases**	**Rs 743 (Absolute Minimum Cost)**

Managerial Impact: Reallocating 2 cases along the loop reduced total freight expenditure from Rs 779 down to **Rs 743** (saving Rs 36, exactly 2 × |Δ22| = 2 × 18 = 36). Re-evaluating MODI shows all $\Delta ij \geq 0$, proving that Rs 743 is the global mathematical optimum.

Chapter 6: Comparative Performance Analysis of NWCR, LCM, and VAM

6.1 Step-by-Step Initial Solution via North West Corner Rule (NWCR)

To appreciate why advanced optimization algorithms are indispensable in corporate logistics, let us solve the exact same 3-Factory by 4-Warehouse consumer goods problem using the purely positional North West Corner Rule:

1. **Cell (P1, W1):** Allocate $\min(a1 = 7, b1 = 5) = 5$ cases. W1 demand satisfied (cross Col W1). P1 remaining = 7 - 5 = 2 cases. Cost = 5 × 19 = Rs 95.
2. **Cell (P1, W2):** Step right to (P1, W2). Allocate $\min(a1 = 2, b2 = 8) = 2$ cases. P1 exhausted (cross Row P1). W2 remaining = 8 - 2 = 6 cases. Cost = 2 × 30 = Rs 60.
3. **Cell (P2, W2):** Step down to (P2, W2). Allocate $\min(a2 = 9, b2 = 6) = 6$ cases. W2 satisfied (cross Col W2). P2 remaining = 9 - 6 = 3 cases. Cost = 6 × 30 = Rs 180.
4. **Cell (P2, W3):** Step right to (P2, W3). Allocate $\min(a2 = 3, b3 = 7) = 3$ cases. P2 exhausted (cross Row P2). W3 remaining = 7 - 3 = 4 cases. Cost = 3 × 40 = Rs 120.

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5. **Cell (P3, W3):** Step down to (P3, W3). Allocate min(a3 = 18, b3 = 4) = 4 cases. W3 satisfied (cross Col W3). P3 remaining = 18 - 4 = 14 cases. Cost = 4 × 70 = Rs 280.
6. **Cell (P3, W4):** Step right to final cell (P3, W4). Allocate min(a3 = 14, b4 = 14) = 14 cases. Both P3 and W4 satisfied! Cost = 14 × 20 = Rs 280.

Total Initial NWCR Transportation Cost:

> $$Z_{NWCR} = (5 \times 19) + (2 \times 30) + (6 \times 30) + (3 \times 40) + (4 \times 70) + (14 \times 20) = 95 + 60 + 180 + 120 + 280 + 280 = \text{Rs } 1,015$$

Legend: Z_NWCR = Aggregate freight expenditure under North West Corner Rule.

6.2 Step-by-Step Initial Solution via Lowest Cost Method (LCM)

Now, let us solve the same distribution problem using the greedy Lowest Cost Method (Matrix Minimum):

1. The absolute lowest unit cost in the entire matrix is **Rs 8 at cell (P3, W2)**. Allocate min(a3 = 18, b2 = 8) = 8 cases. W2 satisfied (cross Col W2). P3 remaining = 18 - 8 = 10 cases. Cost = 8 × 8 = Rs 64.
2. Next lowest cost among remaining cells is **Rs 10 at cell (P1, W4)**. Allocate min(a1 = 7, b4 = 14) = 7 cases. P1 exhausted (cross Row P1). W4 remaining = 14 - 7 = 7 cases. Cost = 7 × 10 = Rs 70.
3. Next lowest cost is **Rs 20 at cell (P3, W4)**. Allocate min(a3 = 10, b4 = 7) = 7 cases. W4 satisfied (cross Col W4). P3 remaining = 10 - 7 = 3 cases. Cost = 7 × 20 = Rs 140.
4. Next lowest cost is **Rs 40 at cell (P3, W1)**. Allocate min(a3 = 3, b1 = 5) = 3 cases. P3 exhausted (cross Row P3). W1 remaining = 5 - 3 = 2 cases. Cost = 3 × 40 = Rs 120.
5. Next lowest cost among active cells of Plant P2 is **Rs 40 at cell (P2, W3)**. Allocate min(a2 = 9, b3 = 7) = 7 cases. W3 satisfied (cross Col W3). P2 remaining = 9 - 7 = 2 cases. Cost = 7 × 40 = Rs 280.
6. Finally, allocate remaining 2 cases of P2 to remaining cell **(P2, W1)** with cost Rs 70. Allocate 2 cases. W1 satisfied! Cost = 2 × 70 = Rs 140.

Total Initial LCM Transportation Cost:

> $$Z_{LCM} = (8 \times 8) + (7 \times 10) + (7 \times 20) + (3 \times 40) + (7 \times 40) + (2 \times 70) = 64 + 70 + 140 + 120 + 280 + 140 = \text{Rs } 814$$

Legend: Z_LCM = Aggregate freight expenditure under Lowest Cost Method.

6.3 Comprehensive Economic Comparison Matrix

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Optimization Stage / Method	Total Logistics Cost (Rs)	Variance from Global Optimum	Managerial Efficiency Rating
North West Corner Rule (NWCR)	Rs 1,015	+ Rs 272 (+ 36.6% excess cost)	Extremely Poor (ignores economic parameters)
Lowest Cost Method (LCM)	Rs 814	+ Rs 71 (+ 9.6% excess cost)	Moderate (greedy local optimization)
Vogel's Approximation Method (VAM)	Rs 779	+ Rs 36 (+ 4.8% excess cost)	Outstanding (evaluates penalty regrets)
MODI Global Optimal Solution	**Rs 743**	**Rs 0 (0.0% — Global Mathematical Minimum)**	Absolute Global Optimum

Chapter 7: Practical Degeneracy Resolution and Unbalanced Systems

7.1 Detailed Mechanics of the Epsilon (ϵ) Protocol

To demonstrate how degeneracy is resolved when total allocations are strictly less than $m + n - 1$, consider a supply network where Plant 1 has capacity 20 and Plant 2 has capacity 30, distributing to Warehouse 1 (demand 20) and Warehouse 2 (demand 30). Allocating 20 units to (P1, W1) simultaneously satisfies Plant 1 supply and Warehouse 1 demand in a single step! Allocating 30 units to (P2, W2) completes the schedule.

However, total allocations = 2, whereas $m + n - 1 = 2 + 2 - 1 = 3$ allocations are strictly required! The solution is degenerate by 1 basic cell. If an analyst attempts to calculate dual variables (u_i, v_j), the system of equations breaks down.

The Epsilon Insertion Procedure:

1. Identify all unallocated cells that do NOT form a closed loop with existing basic cells (P1-W1 and P2-W2). Both (P1, W2) and (P2, W1) are independent.
2. Compare the unit costs of these candidate cells. Select the cell with the lower shipping cost (say, cell P1-W2).
3. Assign the infinitesimal quantity $\epsilon > 0$ to cell (P1, W2). The basic cells are now: (P1, W1) = 20, (P1, W2) = ϵ, and (P2, W2) = 30. Total basic allocations = 3 $(m + n - 1)$!
4. The dual equations can now be solved cleanly: $u_1 + v_1 = c_{11}$, $u_1 + v_2 = c_{12}$, and $u_2 + v_2 = c_{22}$. The opportunity costs (Δ_{ij}) for all remaining empty cells can be audited without mathematical error.

7.2 Handling Unbalanced Systems with Shortages and Penalties

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In many commercial transportation environments, unsatisfied customer demand is not merely a harmless omission; it triggers contract cancellation penalties, customer goodwill erosion, or emergency third-party subcontracting expenses. When corporate planners formulate an unbalanced transportation model with excess demand ($\sum bj > \sum ai$), instead of assigning zero shipping costs to the Dummy Plant, they frequently assign **per-unit stockout penalty costs** to the dummy origin cells ($cdummy,j = Penaltyj$).

When VAM and MODI are applied to this penalty-weighted matrix, the algorithm strategically allocates the supply deficit to destinations with the lowest financial penalties, shielding high-margin, high-penalty strategic clients from delivery disruption.

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